The Goldbach Conjecture demonstrated

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Jan Bauwens, Serskamp, 2005

D/2005/Jan Bauwens, editor

NUR: 918

ISBN: 90-77532-07-2

Original title: *Het Vermoeden van Goldbach. Een Bewijs*

(Tweede, aangevulde uitgave - ISBN 90-77532-07-2)

Preface and contents

The Conjecture named after the German mathematician, Christian Goldbach (1690-1764), and stating that each even number bigger than *2* can be written as the sum of two prime numbers, was an unsolved riddle until today. At least this is what has always been claimed. By the means of this work, its author contradicts this assertion.

This demonstration arose from the intuition that the Goldbach problem cannot be solved but on the condition that the decomposition of numbers can be represented simultaneously into terms and into factors.

So, besides the algebraic demonstration, the author brings the specific representations of numbers, on which both the kinds of decomposition have been made visible at once.

This translation of the second edition of the booklet: *"Het Vermoeden van Goldbach. Een bewijs"*, contains three mutually undependent paragraphs. The first paragraph, being the most important one, introduces the algebraic demonstration of the thesis known as *the Goldbach Conjecture*. The second paragraph as well as the third one present each of them a more intuitive approach to the Goldbach problem, aiming to introduce a (not algebraic) 'logical' insight in the unassailableness of the truth expressed in Goldbach's intuition.

The contents of this booklet are as such:

Serskamp, 2004-2005

The Goldbach Conjecture demonstrated

§1. The algebraic demonstration

Goldbach's to be proven: *"Each even number E that has a value of at least* 4*, can be written as a sum of two prime numbers."*

> (!) How do we proceed in this proof? We give an indirect demonstration. This means that our demonstration has the following structure: IF Goldbach were untrue, THEN a contradiction would result from it.

Proof: <u>Suppose Goldbach's conjecture were untrue</u>. Out of this follows the existence of at least one even number *E*, so that it happens that in the equation $p_i+(E-p_i)=E$ (°), the term $(E-p_i)$ will always be a compound number (p_i representing irrespective which prime number that is smaller than *E*).

> (!) Because Goldbach says that each even number that is at least *4*, can be written as a sum of two prime numbers, the specific supposition that Goldbach's conjecture were untrue, implies the existence of at least one even number *E* being bigger than *2*, which cannot be written as a sum of two prime numbers. So: how will this even number *E* look like? It will be a number *E*, so that all prime numbers smaller than *E* will have to be added up with a compound number in order to produce *E*. The name we provide for all the

mentioned prime numbers smaller than *E*, is *p*. So, the number *p* is a variable number, which means that *p* can have each value that generates sollutions for our equation "(°)". For a reason which will become clear further on, we provide *p* of an index, and so, from now on we will spreak of p_i. By the means of this index, we do not intend to give concrete form to *p*, we only intend to specify *p* a little more in case of necessity.

So, if Goldbach were untrue, than we would have the certainty that the numbers that have to be added up with whatever number *p* in order to get *E*, and these are all the numbers $(E\text{-}p_i)$, otherwise called: the 'complements' of all numbers p_i , would be compound numbers. Stating that these numbers are compound, means that they are being composed out of at least two prime number factors. For this reason we provide these numbers (E-p_i) with the name mp_j . From this follows that (E-p_i)=mp_j . In this name, as has been said, p_j refers to one of the "at least two" mentioned prime number factors out of which mp_j must be composed, and so: out of which each one of the 'complements' of the prime numbers smaller than E must be composed. The second prime number factor is hidden in the factor m. This means that m is a natural number bigger than 1. Possibly m contains either thousands of mutual different prime number factors, or thousands of the same of them, or, still otherwise, that it contains only once certain prime number factors, while it frequently contains others of them, or, still otherwise, it means that either p_j is frequently a part of m or that this is not the case. We leave alone all these possibilities because they are of no relevance concerning our demonstration. The only relevant claim that we must lay on m, is this one: m must be natural and must be bigger than 1.

So this equation will be sound: $(E-p_i)=mp_j$, m being a natural number bigger than 1; and p_i and p_j , being both prime numbers, not necessarily mutually different.

(!) Thus we demand that p_i en p_j should be mutually distinguished. We know both of them being prime numbers and at once being variables, but in the following statements the relevance of the specification added by means of the subjoined index may become clear: more specifically, we will distinguish between two possibilities.

For those are the two possibilities: either p_i en p_j are always (*) mutually equal, or they always (*) mutually differ.

(!) One could put the question because of what reason it is certain that a third possibility - immediately mentioned - can be excluded, namely: supposed cases in which repeatedly a given p_i would equal a certain p_j , and also supposed cases in which repeatedly a given p_i would differ from a given p_j. That such a third possibility can indeed be excluded, is been proven in the paragraph indicated as "(*)", at the end of this very demonstration.

Suppose case 1: $p_i=p_j$. Hence it follows out of $(E-p_i)=mp_j$ that $(E-p_i)=mp_i$ and that $E=mp_i+p_i$ and that $E=p_i(m+1)$. [We know that: $m>1$, and so we know that $m+1$ equals at least 3; but we also know that $p_i(m+1)$ must be even, so we do know that $(m+1)$ must be even, and that m is at least 3 and is always odd. But this is irrelevant]. In this case, the equation $E=p_i(m+1)$ says: "E is a multiple of p_i ; E contains the factor p_i."

> (!) In other terms: if we might exchange mutually p_i en p_j, then we know that p_i is a factor of E. But because p_i represents whatever prime number smaller than E, it

is impossible for E to exist under the mentioned circumstances, because of the fact that a number E that contains all prime number factors that are smaller than E itself, never can be big enough to do so. From this must be concluded that this first case, in which is stated that $p_i = p_j$, leads to a contradiction. In still other terms: in this first case Goldbach cannot be untrue.

If the reader were not convinced of what we stated above, namely that p_i represents whatever prime number that is smaller than E, and that E, consequently, must contain all prime number factors that are smaller than E itself, (so that E, under these circumstances, cannot exist because an E that contains all prime number factors smaller than E itself, can never be big enough to do so), we must say the following:

We must realize well that p_i in this very case represents each possible prime number smaller than E. Because, supposing that we should exclude or forget only one single p_i (this is: one of all prime numbers smaller than E), we should deny our indirect demonstration, our demonstration 'ex absurdum'. For, concerning the first case (in which has being supposed that $p_i=p_j$), this statement namely says this: if we substract each one, the one after the other, of the prime numbers smaller than E, from E, then each substraction generates compound numbers, among which numbers composed out of the prime number itself that is been substracted in that very case. Let us explain this with an 'example': E-$2=2m$; E-$3=3m$; E-$5=5m$; E-$7=7m$; E-$11=11m$... (E is a constant; m is a variable; p_i is, one case after the other and case by case, each prime number smaller than E, (beginning with the number 2)). One sees clearly that, by this, it is been demonstrated that in this case *each* p_i *smaller than* E, will be at the same time a *divisor* of E.

Suppose the second case: p_i *verschilt van* p_j.

(!) This case can bring up some problems of interpretation, so some explanation will be necessary. Our stating that p_i always must differ from p_j , more especially in the equation $(E\text{-}p_i)=mp_j$, does not mean that p_i en p_j should represent well-determined numbers - at the contrary: they stay variables, just like before, but this very time we demand that they never equal each other - albeit case by case. In this way, all the following cases are being excluded: *E-2=m.2*, *E-3=m.3, E-5=m.5, E-7=m.7, E-11=m.11*, etcetera. Suppose that the number *30* were a certain number *E* that should contradict the Goldbach Conjecture, then our second case now signifies that all cases of our equation in which the prime number factors *2*, *3* en *5* are representing either p_i or p_j , must be excluded, precisely because they contradict the presupposition of this case, namely that p_i en p_j must differ mutually. E.g.: in the mentioned case, the number ***2*** is been excluded from participation because in *30-**2**=14.2* it holds that $\boldsymbol{p_i}$ equals $\boldsymbol{p_j}$. E.g. the number ***3*** is been excluded from participation because in *30-**3**=9.**3*** it holds that $\boldsymbol{p_i}$ equals $\boldsymbol{p_j}$. E.g. the number ***5*** is been excluded from participation because in *30-**5**=5.**5*** it holds that $\boldsymbol{p_i}$ equals $\boldsymbol{p_j}$.

Hence it follows from $(E\text{-}p_i)=mp_j$ that p_j can never be a factor of *E*.

(!) This conclusion will be demonstrated immediately. Now, let us make clear why we do make this conclusion: we do so because, being able to assure ourselves that the numbers p_j can never be factors of *E*, we also know that the numbers p_i are the unique factors of *E*, and this is the case because of the fact that if p_i en p_j represent all prime numbers smaller than *E* -

and, indeed, under the condition that they are been mutually distinguished, they do so -, then there are no more prime numbers left that are smaller than E, apart from p_i en p_j. If, in addition, we know that the factors p_i are the unique prime number factors in E, then we know that the cases in which p_j and p_i differ mutually, do not exist, and so we know that our second case cannot exist.

The proof:

(!) Let us prove that a number p_j that differs from p_i can never be a factor of E. This is an indirect proof, a demonstration 'ex absurdum': we namely suppose that p_j were a factor of E and, out of this supposition, we can see arising a conclusion that contradicts our presuppositions.

Suppose that p_j were a factor of E, and so that it would hold that $E=bp_j$, b being a natural number bigger than 1, then it follows from $(E\text{-}p_i)=mp_j$ that holds: $bp_j\text{-}p_i=mp_j$, out of which follows that $bp_j\text{-}mp_i=p_i$, and that $(b\text{-}m)p_j=p_i$. And this would signify: either that p_i were a multiple of p_j [namely as $(b\text{-}m)>1]$, which were impossible because both of them are prime numbers; or that $p_i=p_j$ [namely as $b\text{-}m=1$], which takes us back to the first case; or that $p_i=p_j=0$ [namely as $b=m$], which would cause E to equal 0 or 2 [but here must be noticed that m is odd while b

should be even because we stated that $E=bp_j$, so that $b=m$ is impossible]. So far this proof. The second case shows that in the equation $(E-p_i)=mp_j$, in which p_i en p_j differ mutually, E can never contain any other prime number factor besides p_i , except the even prime number factor 2 [in this very case $(E-p_i)=mp_j$ becomes: $2-2=0$].

(!) Concerning the equation $(b-m)p_j=p_i$ one could be tempted to suppose the deceitful possibility that, apart from p_i en p_j , still other prime numbers could be in the game, namely specific prime numbers that differ both from p_i and p_j, which nevertheless would be factors of E. This misconception could appear easily with regards to (1°) the temptation of the imagination of concrete examples and, (2°), the possibility that one should forget that this very reasoning does hold under the specific supposition that the Goldbach Conjecture were untrue. The combination of the two misconceptions mentioned, easily could give way to the doubt of our nevertheless formally demonstrated conclusion. First of all, let us concretise by the means of what thoughts such a doubt could arise.

E.g. one could imagine a concrete even number, such as the number *30*, and then he could suppose that Goldbach's Conjecture were contradicted by the case $E=30$. One could think of a concretisation concerning our equation as follows:

Our equation says: E-p_i=mp_j . We take E to be *30*, and p_i to be *2*. In this case we can, e.g., substitute m by *4*, and so p_j becomes *7*, and so we get: *30-2=4.7*. In doing so, one could wrongly conclude that, *7* not being a factor of *30*, while, e.g. the prime numbers *2*, *3* and *5* yet being so! Now what is wrong in this way of thinking? This reasoning fails because in it, it appears that one seems to have forgotten that we have been reasoning under the supposition that p_i differs from p_j. More explicitly, we can easily contradict the misleading example by indicating the fact that in all those cases in which one of the mentioned prime number factors (*2*, *3* of *5*) appears, p_i necessarily equals p_j. Let us explain these cases to make sure:

in *30*-**2**=*14*.**2** it holds that $\underline{p_i}$ equals p_j ;

in *30*-**3**=*9*.**3** it holds that $\underline{p_i}$ equals p_j ;

in *30*-**5**=*5*.**5** it holds that $\underline{p_i}$ equals p_j .

Hence we may conclude: *2*, *3* and *5* are indeed factors of *30*, but in all those cases wherein they generate sollutions to our equation, they do not act up to the demand that (in that very case) p_i and p_j should differ mutually. And this is precisely the demand that constitutes the second case in which we are reasonning here.

Obstinated sceptics however, nevertheless they do not succeed in making a formal counter-proof of our statements, could resist all evidence and say that the prime number factors *2*, *3* en *5* are once and for ever factors of the number *30*. Well, to free them from these misconceptions, we can at last indicate the following: the number $E=30$ from the deceitful example cannot exist... precisely because, p_j differing from p_i , p_j never can be a factor of E ("*30*").

Conclusion:

p_i and p_j being mutually different, p_j cannot be a factor of E, as has been demonstrated above. It is also clear that, in that very case, p_i cannot be a factor of E either. For in the second case, p_i is not a factor of $E-p_i$, and consequently it is not a factor of E either.

(!) We will now make more explicit the reason why p_i is not a factor of E: suppose that p_i were a factor of E. Then it follows that p_i is a factor of E-p_i. Then p_i is a factor of m because E-p_i=mp_j. Now suppose that m=np_i, then it holds that E-p_i=(np_i)p_j. This can be written otherwise as follows: E-p_i=(np_j)p_i. In this, m=np_j, and so it holds that: E-p_i=mp_i. Yet we had supposed that p_i should differ from p_j, and this is not the case, because here we are dealing again with the first case. So p_i cannot be a factor of E. Again: m may contain all kind of factors, but it may never contain p_i, for in that case we could take p_i out of it and put p_j into it and, in doing so, we would get a form that only holds in our first case.

Hence it may be concluded that, in the second case, nor p_j nor p_i can be a factor of E. Though it is given that p_i and p_j are prime numbers.

If the Goldbach Conjecture were untrue, then it holds that, in the equation $(E\text{-}p_i)=mp_j$, the prime number factors p_i en p_j necessarily equal each other. But in that very case, E is a multiple of p_i. Now p_i represents each possible prime number smaller than E. So E should contain all prime number factors that are smaller than E itself in order the Goldbach Conjecture to be untrue. But because of the statement saying "that there exists at least one prime number factor between each number and its twofold" (— being Bertrands postulate, proved as a statement by Tscheby-cheff), it can be demonstrated easily that E never can be big enough to fulfil this condition. So the Goldbach Conjecture can never be untrue. Which was to be proven.

> (!) We remember: in the first case, the supposition that it always holds that $p_i = p_j$, leads to the conclusion that the Goldbach Conjecture cannot be untrue; in the second case, the supposition that p_i always differs from p_j , leads to a contradiction, and so leads to the same conclusion. Out of this we may conclude as follows: whatever case we ever choose, each time we must conclude that the Goldbach Conjecture cannot be untrue.

(*) Now we shall demonstrate that this is "always" the case, and so: that it can never be that at one time these prime numbers should equal each other while at another time they should mutually differ. Suppose namely that in $(E\text{-}p_i)=mp_j$, p_i differs from p_j, then, as is been demonstrated above, it is impossible to E to contain the factor p_j , and consequently E is not a multiple of p_j ; suppose, at the contrary, in $(E\text{-}p_j)=np_k$, being $p_j=p_k$, then E must contain the factor p_j ; now, because the implicanda of the two suppositions made are mutually contradictory, it results that these presuppositions cannot be made. So we can conclude: either it holds in each case that, in the equations of the form $(E\text{-}p_i)=mp_j$, the involved prime numbers at the left hand side and at the right hand side of the equationmark are mutually equal, or it holds in each case that they mutually differ, but for sure it never can hold that they should equal one another at one time and differ one from another at another time, because from the moment on that such a case should appear, a contradiction would follow. By this, "(*)" is been demonstrated.

Let us now make the last paragraph "(*)" from above somehow more explicit.

The statement "(*)" is as such:

If the "complement" of a prime number p_i smaller than E can be written as a compound number that contains a prime number factor p_j , being different from the given prime number p_i , then this p_j cannot be a factor of its own "complement".

(Here "the complement of p" indicates: "*E*-p").

Formally: if $E\text{-}p_i=m'p_j$ (p_i and p_j being mutually different), then not $E\text{-}p_j=m''p_j$. And then $E\text{-}p_j$ has not a single common factor with *E*. (***)

<u>Proof</u>: we will demonstrate that $m''p_j$ contains not a single factor of $E\text{-}p_j$ (because $m''p_j=E\text{-}p_j$): it has already been proven that p_j cannot be a factor of *E*. We now demonstrate also, by a "reductio ad absurdum", that *m''* cannot contain any factor of *E*: suppose namely dat *m''* would contain a factor of *E*, so that $m''=f.n$, then we substitute this in $E\text{-}p_j=m''p_j$ and we get: $E\text{-}p_j=f.n.p_j$. But then it follows that $E=f.n.p_j+p_j = p_j(f.n+1)$: as we

can see, in this case also p_j would be a factor of *E*, what already has been excluded. Hence also *m''* does not contain a factor of *E*.
So we do not exclude that in the case underlined above, this "complement" **can** also be written as $m'p_i$ (as it could be also the case e.g. with $E=50$, $p_i=5$, $p_j=3$, $m=15$ *and* $m'=9$, hence: $50\text{-}5=15.3=9.5$. But do remark here that the reason why *50-5* can also be written as *9.5*, lays in the fact that p_j, being *3*, is a factor of *m*, being *15*), but we can exclude these cases because they are not relevant in our demonstration, as will be proved further on, in the paragraph "(£)". It is important in that case that we recognise the following:
If, with respect to **a certain prime number** p_i, the second case appears, which means: if

E-p_i=mp_j , p_i being *different* from p_j, then **no other prime number** p_j can appear - a prime number p_j so that E-p_j=mp_k , p_j being *equal* to p_k. (°). Proof: suppose that E-p_i=mp_j , p_i being different from p_j, then p_j cannot be a factor of E (because of (***)), nor of E-p_j. (Let us apply here the given example once again: *if 50-5 can be written as 15.3, then 50-3 can never be written again as a number containing the factor 3*).

So let us explain why this suffices in the demonstration:

If the Goldbach Conjecture were untrue, then we had to demand the "complement" of EACH prime number smaller than E to be compound.

Now we have distinguished between two cases IN THE FORMULA: E-p_i=mp_j, namely a first case wherein p_i and p_j always mutually equal, and a second case wherein p_i en p_j always mutually differ (£). We remember that in these, p_i en p_j were not any concrete values of prime numbers: they represented EACH prime number smaller than E, albeit with the specific limitations due to the respective cases. The fundament of these method is the thesis "(*)", which states that the first and the second case can never appear at the same time. So what does this mean?

(£) What is being meant by "being always mutually equal" and "being always mutually different"?

(1°) "always mutually equal": By saying that, in the equation $E\text{-}p_i\text{=}mp_j$, the factors p_i en p_j always mutually equal, is been meant that all cases are been excluded wherein p_i and p_j mutually differ. This means that we exclude all cases wherein the equation can be written as $E\text{-}p_i\text{=}mp_j$, p_i and p_j being different. Hence we exclude all cases wherein the variable *m* should contain a factor p_j different from p_i. Hence we exclude all cases wherein a factor p_j different from p_i should be hidden into *m*. Still otherwise said: we forbid *m* to contain a factor p_j different from p_i.

(2°) "always mutually different": By saying that, in the equation E-p_i=mp_j, the factors p_i en p_j always mutually differ, is been meant that all cases are been excluded wherein p_i and p_j mutually equal. This means that we exclude all cases wherein the equation can be written as E-p_i=mp_j , p_i and p_j being equal. Hence we exclude all cases wherein the variable m should contain a factor p_j equal to p_i. Hence we exclude all cases wherein a factor p_j equal to p_i should be hidden into m. Still otherwise said: we forbid m to contain a factor p_j equal to p_i .

The crucial question left in these is as a matter of fact this one: do we not forget a number of "specific cases", namely those cases wherein yet prime numbers are hidden into *m* which, respectively the first and the second case, differ from or equal p_i ?

Well, what is really stated in the paragraaf "(*)", is this: respecting the presupposition under which we are reasonning (namely that the Goldbach Conjecture were untrue), and so respecting the complement of each prime number to be compound, the "specific cases" mentioned above cannot arise. It is easy to demonstrate this: we only have to demonstrate that out of $E\text{-}p_1 = m'p_2$ never can be concluded that $E\text{-}p_2 = m''p_2$, and never that $E\text{-}p_3 = m'''p_3$, $E\text{-}p_4 = m''''p_4$, etcetera. Once this has been proved, it has been proved that these "specific cases" are excluded here and that the division of the problem into both the proposed cases, is a sound and a justified one. This demonstration follows in the paragraph beginning with the words: "Here at least...".

This means: if we find a specific case (here: a concretised value to the prime number p_i, and also to **a certain** p_i) wherein E-p_i=mp_j , p_i being different from p_j, then we will never again find **another** prime number p_j so that E-p_j=mp_k and p_j=p_k. For the finding of an other prime number p_j=p_k, and thus any p_j , being a factor of E-p_j and consequently of E, would result into a contradiction with the presupposed.

In other terms: if we find a specific case (here with a concretised value to the prime number p_i, and thus to **a certain** p_i) so that

E-p_1=mp_2 , p_1 being different from p_2, then we will never again find **another** prime number p_2, p_3, p_4, etcetera. so that E-p_2=mp_2, E-p_3=mp_3, E-p_4=mp_4, etcetera. For the finding of another prime number p_2, or thus a p_2 , being a factor of E-p_2 (idem concerning p_3, p_4, etc.) and thus of E, would result into a contradiction with the presupposed.

Let us illustrate this: suppose that the searched p_j would exist (we already know about it that it is a prime number smaller than E), then also its "complement", being E-p_j, should have to be compound in order to fulfil our presupposition (namely: that Goldbach were untrue). If then we should accept that we got an equation of the form E-p_j=mp_k , p_k being equal to p_j , then in this case p_j would be a factor of E. But this case is already been excluded by the thesis "(***)".

Here are some concrete examples:

If E-2=m'' . 3, then not E-3=m''' . 3;

If E-3=m''' . 5, then not E-5=m''''' . 5;

If E-2=m'' . 7, then not E-7=m''''''' . 7; etcetera.

HERE AT LEAST THE FORMAL EXCLUSION OF THE LAST POSSIBLE OBJECTION:

The case one still could throw up, is this one:

Suppose E-p_1=$m'p_2$ as well as E-p_3=$m'''p_3$ were the case. (°°°)

We know:

(1°) **If $E\text{-}p_1=m'p_2$ then never $E\text{-}p_2=m''p_2$ and then p_2 is not a factor of E (°). But in that case it also holds that m'' contains not one factor of E** (due to "(***)"). We repeat the proof: suppose that m'' contains a factor of E, so that $m''=f.n$, then we substitute this in $E\text{-}p_2=m''p_2$ and so we get that $E\text{-}p_2=f.n.p_2$. But then it holds that $E=f.n.p_2+p_2 = p_2(f.n+1)$: as one can see, in this very case also p_2 would be a factor of E, what already has been excluded. So also m'' does not contain any factor of E.

(2°) **If $E\text{-}p_3=m'''p_3$ then never $E\text{-}p_2=m''p_3$.**

Proof: We know: _if $E\text{-}p_2=m''p_3$ then never $E\text{-}p_3=m'''p_3$_ [Here p_2 and p_3 refer respectively to p_i and p_j of the general rule (see the bold text at the beginning of this demonstration) that says: *if* $E\text{-}p_i=m''p_j$ *then never* $E\text{-}p_j=m'''p_j$]. Due to logical reasoning: *(if A then not B) is equivalent with (if B then not A)*, we can rewrite the (underlined) implication from above into the following one: if $E\text{-}p_3=m'''p_3$ then never $E\text{-}p_2=m''p_3$.

Now the question is: does a contradiction follow from this?

Here is the answer:

yet given are the four following cases:

$E\text{-}p_1=m'p_2$ (1) and

$E\text{-}p_3=m'''p_3$ (2) and

not $E\text{-}p_2=m''p_2$ (3) and

not $E\text{-}p_2=m''p_3$ (4).

The second case "(2)" tells us that p_2 is not a factor of E.

However E-p_2 must be compound (because the Goldbach Conjecture is supposed to be untrue in here).

Suppose E-p_2=$m''p'$, p_2 differing from p', due to (3). (****)

Due to the given "(1)", namely: E-p_1=$m'p_2$, we know that E-p_2 contains not one factor of E (see also "(°)"). For from E-p_1=$m'p_2$ follows that E-p_2 and E have not one common factor.

So, $m''p'$ [for it equals E-p_2, due to what is supposed in "(****)"] contains not one factor of E. ($)

We now transport the term p_2 in the equation "(****)" to the right-hand-side part of the equation and so we get: $E=m''p'+p_2$. Here we remark again that p' differs from p_2 and that $m''p'$ contains not a single factor of E (due to"($)"), and consequently m'' contains not a single factor from E.

But now also $m''p'+p_2$ must be compound, because E is so.

We know that m'' contains not a single factor of E [due to "(***)"] and consequently also not of $E-p_2$. Moreover p_2 is not a factor of E. Due to "(3)" it holds that: if $E-p_2=m''p_2$, then $(E-p_2):p_2=m''$, m'' being a not natural number. If supposing now that **$E-p_2=m''p'$ [see: "(****)"], then m'' must contain the fac-**

tor p' in its denominator, and then we can write: $E\text{-}p_2=(M:p').p'$, M being a natural number. Then it holds that $E\text{-}p_2=M$. Then p' is not a factor of $E\text{-}p_2$, and then $E\text{-}p_2=m''p'$ is impossible. Then $E\text{-}p_2=M$ is not compound. To check up: suppose M to be compound, and suppose $M=N.q$, q being of course not a factor of $E\text{-}p_2$, then it would hold that: $E\text{-}p_2=N.q$, N being not compound. Again to check up: suppose N to be compound, then it holds that $N=R.r$, r being of course not a factor of $E\text{-}p_2$, then it would hold that: $E\text{-}p_2=R.r.q$, R being not compound. In this way we can extract, out of our original M, all putative factors, yet being **attentive** to the fact that none of them ever can be a factor of E-p_2, and so the equation cannot become sound for whatever value. So E cannot be compound, and can only have the value *2*. From this contradiction follows that the presupposed "(°°°)" is impossible.

Jan Bauwens, 19 juni 2004.

§2. An intuitive demonstration

Remark

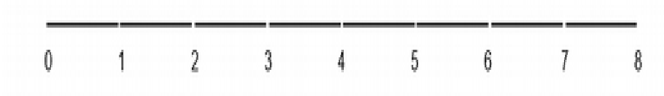
The demonstration in §1 arose from the intuition that Goldbach's problem could not be solved but on the condition that the decomposition of numbers could be represented in terms as well as in factors, and that this could be done at once. So the original attempt to prove the Goldbach Conjecture had an intuitive character. The elimination of informal aspects by the formal responding of possible objections, induced the formal, algebraic prove that has been exposed in de first paragraph of this booklet.

In this intuitive demonstration we will proceed by the means of an example. We do so to allow a good understanding of the demonstration. The general approach follows at the end of this exposition.

Goldbach says that each even number bigger than *2*, can be written as being the sum of two prime numbers.

Let us take an arbitrary number, e.g. the number *8*. The mentioned thesis only holds in the case of an E so that $E>8$, yet this is no object to our exposition. As we want to keep our representation as simple as is possible, we must ask the reader to have some patience: the general approach will follow later on.

We now represent the number *8* as follows:

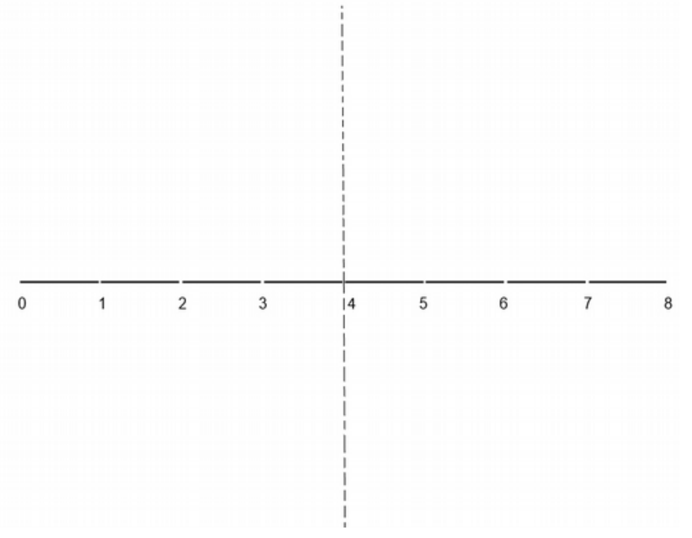


The reason why we will represent the numbers from now on in the way shown here, must be clear: we will have to be able to approach each number as being a unity that, in a specific number of ways, can be composed out of different *terms* at the one hand, and out of different *factors* at the other hand. This is because the Goldbach-problem concerns a well-specified relationship between, at the one hand, the terms of even numbers and, at the other hand, their factors. In this way, our manner of representation allows us to observe how the number *8*, represented by a fragment with a length of *8*, is been composed out of the terms *2* and *6*, for we can add mutually these fragments of respectively length *2* and *6*, and, at the same time, we can see how the number *8* is been composed out of factors, e.g. the factors *2* and *4*, specifically as we can see how the product of the factors *2* and *4* generates *8*. In order to be able to observe this well, we will use 'waves'. We first of all must remark that, by this terminology, we do not aim the physical concept of 'waves', as one should normally think: we just use the specific representation of waves for mere didactic purposes. In this way, e.g., this specific representation of the number *8* will show us that *8* contains the factor *2* (in other terms: the number *8* has the number *2* as a divisor), because the 'wave of *2*' crosses the horizontal axis at *8*. In general, the representation by means of 'waves' shows us how each number is a multiple of those prime numbers which have waves crossing the horizontal axis at the position of that very number.

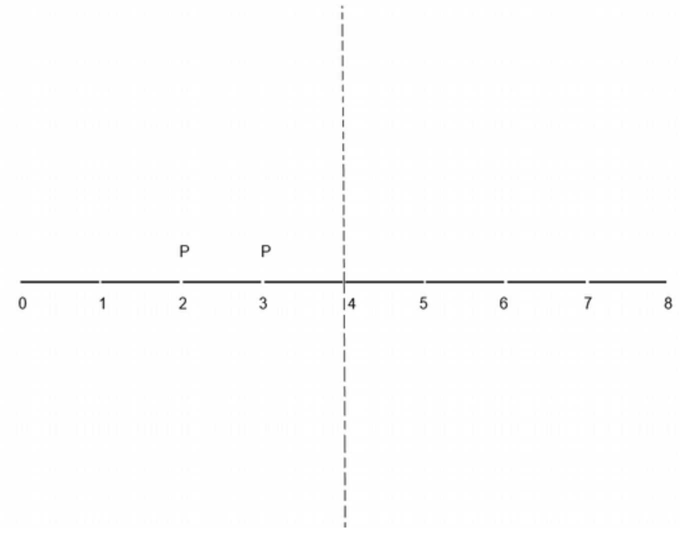
We know that each even number has a natural number as its half. That half can be an even number, an odd number, a prime number or a compound number.

In our example with the arbitrary chosen even number *8*, the half of that number *8* equals *4*.

We now indicate the number *4*, being the half of the number *8*, on our representation of the number *8*, and we do so by the drawing of a dotted perpendicular line on the axis that carries our representation of the number *8*, throughout the 'point *4*', as follows:



We now consider all prime numbers that either are smaller than the half of our even number, or that equal this half. So, we consider all prime numbers p_i , so that $0<p_i \leq 4$, being the numbers *2* and *3* in our example. We now indicate these prime numbers by means of the character "P" on our representation of the number *8*, as follows:



Remark: factually, we do not need the prime number *2*, because the 'counterpart' of *2* (namely: the specific prime number that has to be added up with *2* in order to get a sum being the even number that is in question), will be odd, while the sum of an even number (in casu the number *2*) and an odd one (for the 'counterpart' of *2* never can be even again because *2* is the only even prime number) can never generate an even number.

Now, due to the Goldbach Conjecture, it must hold for each even number E, being bigger than *2*, and also for the even number *8* in our example, that this even number can be written as the sum of two prime numbers (each of them being bigger than *2*).

Remark: we leave the prime number *2* into the play in order to protect the simplicity of our example for the time being.

We now do know that the first one of both intended prime numbers (being p_1) which will always equal the number *2*, will be part of the 'first half' of the number *8*, while the second one (being p_2), which will always equal the number (E-*2*), will be part of the 'second half' of the number *8*. In other terms: concerning p_1 it will hold that: $0<p_1\leq 4$ and concerning p_2 it will hold that: $4\leq p_2<8$.

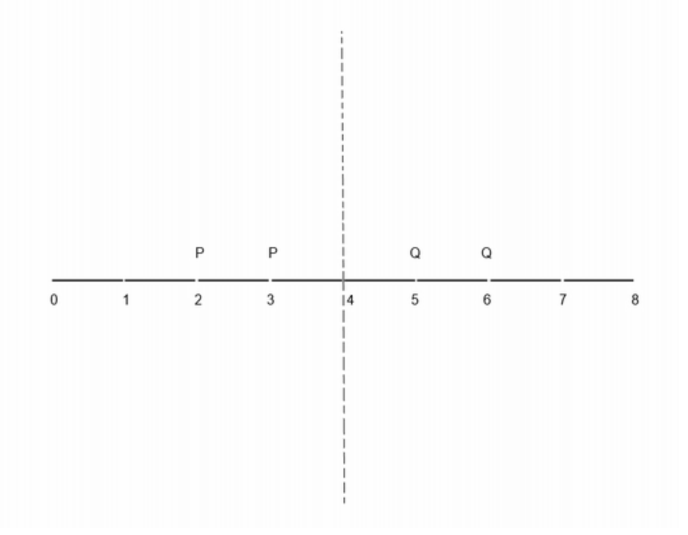
Moreover we do know, as yet has been said, that the sum of both mentioned prime numbers must equal *E*.

We now restrict things to our example, and so we can write: the Goldbach Conjecture means that the number *8* (as well as whatever even number that is bigger than *2*) can be written, either as $2+x_1$, or as $2+x_2$, wherein either x_1 or x_2 is a prime number. (Remark: in these, the numbers *2* and *3* are the prime numbers coming from the first half of the number *8*, and the x_1 and x_2 are numbers coming from its second half - and at least one of these two numbers has to be a prime number in order to consolidate the Goldbach Conjecture).

We now consider, on our representation of the number *8*, the half of *8* (being the number *4*) as a 'mirror'. In general, this mirror equals the number (*E*:*2*).

In doing so, we can observe x_1 (wherein x_1=*E*-*2*) being the reflection (through the indicated mirror) of the number *2*, and x_2 (wherein x_2=*E*-*3*) being the analogue reflection of the number *3*. This holds because we do know that the respective sums of p_i and x_i in both cases must equal *E*.

On our representation of the number *8*, we now indicate these mirror-images by the character "Q", as follows:

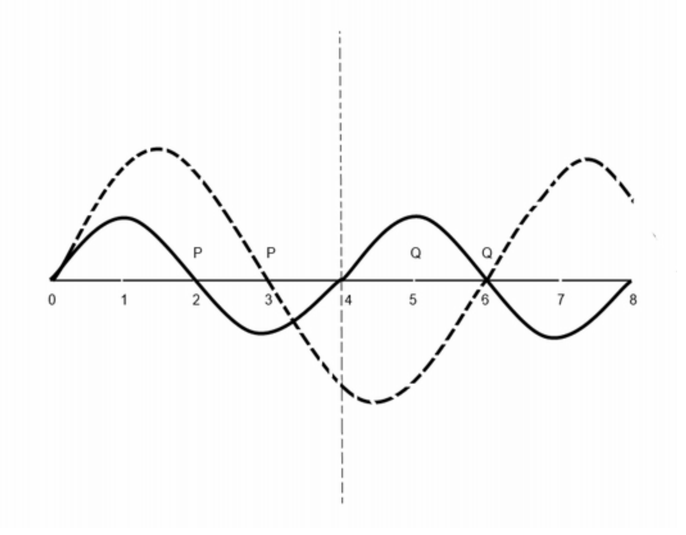


So, what the Goldbach Conjecture expresses, is this: "at least one of the Q's that are been generated in this way, shall be a prime number again (- and this holds for every even number that is bigger than the number *2*)."

At this time, we act as if we did not know which numbers q_i , so that $\leq 4 q_i < 8$, were prime numbers.

We consider again our representation of the number *8*, and we indicate on it all numbers between *0* and *8* which can never be prime numbers; these are well-defined the compound numbers, more explicitly: these are the multiples of the prime numbers out of the first half; so these are the multiples of the prime numbers p_i , so that $0<p_i\leq 4$, which have already been indicated. Let us stress that there exist only two kinds of numbers, being: the prime numbers and the compound numbers. The latter are the multiples of the prime numbers.

We can find these multiples by drawing waves, all of them starting at the point *0* and each wave apart crossing the prime number belonging to it, as follows:



Let us repeat that these 'waves' do not indicate physical waves, for they are only used as a didactic expedience in order to get a clear representation of numbers being composed out of terms and factors simultaneously.

Each wave, originating from *0*, and crossing a specific *P*, factually throws all of its multiples forward as in a whip-lash, and more specifically it gives birth to them repeatedly at each of its crossing-points with the horizontal axis.

Hence in our example we get two waves, namely: (1°) the wave of the prime number *2* (the full line), that indicates the multiples of *2* at each of its crossing-points at the axis and, (2°) the wave of the prime number *3* (the dotted line), that indicates the multiples of *3* at each of its crossing-points at the axis.

In this way we can clearly see:

(1°) that the number *4* cannot be prime due to the wave of *2*;

(2°) that the number *6* cannot be prime due to the wave of *2*;

(3°) that the number *6* cannot be prime due to the wave of *3*;

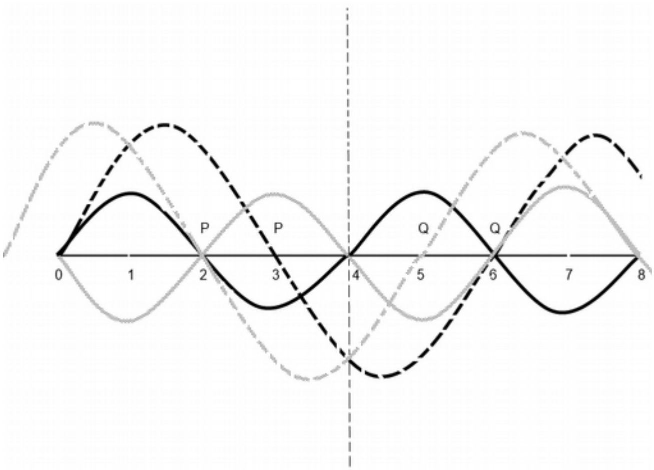
(4°) that the number *8* cannot be prime due to the wave of *2*.

Let us repeat:

all those numbers in the right hand side half of our representation of the number *8*, that are been crossed by one of our prime number waves coming from the left hand side half of that representation, cannot be prime, because it are multiples of prime numbers. Our representation shows us that this is the case concerning the numbers *4*, *6* and *8*, for these are compound numbers.

Let us already remark that all numbers in our right hand sided number half will be prime numbers because there exist no third kind of numbers apart from the compound numbers and the prime numbers.

Now we apply the following manner of representation: in order to integrate the mentioned 'process of mirroring' into our 'wave-method', (in our example concerning the number *8*) we will not only mirror the prime numbers p_i (so that $0<p_i \leq 4$) throughout *4*, but, moreover, we will mirror the waves that have been just generated. When drawing these mirrored waves in red colour, our representation of the number *8* looks as follows:



The waves departing from *0* and going from the Left to the Right hand side (here coloured in grey) will be called 'LRwaves'. The waves departing from *E* and going from the Right to the Left hand side (here coloured also in grey) will be called 'RLwaves'.

As one can see, the LRwaves depart from *0* and, because their mirror-images are the RL-waves, these RLwaves depart from *E*, due to the fact that *E* is the mirror-image of *0*.

Again: the RLwaves are been generated by the mirroring of the LRwaves throughout the mirror (*E*:*2*). [In our representation, (*E*:*2*) equals *4*]. We remark further on that the number *4* is its own mirror-image. We can also see that each number *E* that has a prime number as its half, fulfils the demand of the Goldbach Conjecture. (For that reason, such a number $E=2p_i$ will be excluded as an example in the supposition that follows immediately).

So, here is our 'reductio ad absurdum' (and it is very important to understand this well):
If the Goldbach Conjecture were untrue, then at least one even number should exist in the representation of which the mirror-images of the prime numbers p_i (so that $0<p\leq_{i}4$) would never be prime. In other terms: concerning that number, all mirror-images of the prime numbers p_i out of the first half of our number, would be situated on LRwaves. For the LRwaves always cross the horizontal axis in the second half of our number at points which are multiples of the prime numbers out of the first half of our number.
Though it is clear that this can only be the case on the condition that this even number (- being the number E that would contradict

the Goldbach Conjecture... if it should exist!) were so, that the RLwaves would mirror the LRwaves (because in each case the sum of mirror-images forms the respective even number) — in other terms: if all LRwaves would coincide with the RLwaves.

Firstly, let us remark another thing in order to avoid misunderstandings: the bowing of the mentioned waves, either upwards or downwards, is of no importance for, as yet has been said, in here we do not aim physical waves, but a mere didactic representation; consequently, the waves on the upside of the horizontal axis must be considered as being identical with the waves on the downside of it, as soon as they cross the same points (numbers) on the horizontal axis.

For now, let us suppose that, concerning a given even number *E* being bigger than *2*, all LRwaves would indeed coincide with all RL-waves, then this would mean that the number in question (— and, for now, let us consider the representation of our example of the number *8*) had to contain all prime number factors being either smaller than *4* or equal to *4*.

In general: supposing that, concerning a given even number *E* being bigger than *2*, all LRwaves would indeed coincide with all RL-waves, this would mean that the number in question had to contain all prime number factors being either smaller than (*E*:*2*) or equal to (*E*:*2*). For all these LRwaves, if they are mirrored into RLwaves, will cross the horizontal axis in E; in other terms: they will also 'arrive' in E.

Now we can see the following: to fulfil this condition, for a value of E wherein it holds that $E>8$, this E should have to be bigger than E (sic!), because of the fact that already the product of all prime numbers p_i , so that $0<p_i \leq(E:2)$, is always bigger than E itself, as can be demonstrated easily by the means of Tschebycheff's thesis.

Remark: the cases in which $E8\leq$ as a matter of fact can be handled apart.

So we must conclude that the Goldbach Conjecture cannot be untrue, which was to be demonstrated.

Let us repeat all this briefly:

The Goldbach Conjecture were untrue if an even number E should exist, so that all q_i (being mirror-images of the prime numbers p_i , so that $0<p_i\leq(E:2)$) were multiples (more specifically: multiples of p_i). For in that case no sum consisting of the terms p_i and q_i , both of them being prime, could be found. Now, the numbers situated between $E:2$ and E are either prime numbers, or multiples of prime numbers - there is no third possibility. We know for sure that all multiples are situated on LRwaves which, as we know, throw the multiples of the prime numbers forwards in a whip-lash into the infinite. So we can express this also by saying: the Goldbach Conjecture were untrue if there should exist an

even number E, so that all q_i (being the mirror-images of the prime numbers p_i so that $0<p_i\leq(E:2)$) were situated on the LRwaves (more specifically: if they were either equal to $E:2$ or between $E:2$ and E, which means: if they were situated on the line-fragment $[E:2,E]$), due to the fact that in that case none of these mirror-images would be prime and, consequently, no sum of two prime numbers ever could equal E. For now, no problem would arise if the LRwaves, once beyond $E:2$, would reflect themselves, in this very sense that their forms either at the right hand side or at the left hand side of $(E:2)$ would be the same, for in this case we would know for sure that all q_i would reflect all p_i throughout the mirror $E:2$, and that all these q_i

would be multiples of prime numbers, becau-se in that very case they should be situated on the LRwaves, either at the right or at the left of *E*:*2*. Though the very problem is this: the forms of the waves either at right or at left of *E*:*2* are not necessarily each others mirror-images (and as will be demonstrated, they factually never are, but this we do not know at this very moment). Though, in order all p_i to be reflected in q_i , they yet have to be each other's mirror-images. Well, they could be indeed, namely in the one restricted case in which the forms of the LRwaves situated either between *0* and *E*:*2*, or equal to *E*:*2* (-these are the waves situated on the line-frag-ment [*0*,*E*:*2*]), after habe been mirrored in *E*:*2*, would coincide with the forms of the LRwaves as appearing from the point *E*:*2* on: these are the waves situated on the line-fragment [*E*:*2*,*E*]. So we should try to imagine the existence of an even number *E* (on a representation analogue to the representation of our example) wherein the LRwaves coincide perfectly with their reflections throughout the mirror *E*:*2*, and these reflections have been called RLwaves. In such a representation, all RLwaves will depart necessarily from *E*, because *E* is the reflection of *0*, out of which all LRwaves depart. So, if the LRwaves coincide with the RLwaves, this means that all LRwaves (as has been said: departing from *0*) will arrive at *E*. This means that in that very case, *E* will have to contain all prime number factors p_i . Though, in order this case to be possible, the number *E* (from a value *E*>*8* on) will always be to small: as has been said, this can easily be demonstrated by the means of Tchebycheff's thesis, which states that there is at least one prime number between each number and its twofold. Hence we may conclude that the supposition that would make the Goldbach Conjecture untrue, never can be true itself. What was to be demonstrated.

A complementary approach

Now one could admit that the certainty about the coinciding of the LRwaves and the RL-waves were still unclear: one could say that the existence of common elements of both sets that are constituted respectively by LR-waves and RLwaves, does not necessarily imply the coincidence of these sets. In order to make away with this doubt, we will make still another approach, aiming to show that the coincidence of both sets of waves (being the LRwaves and the RLwaves) necessarily follows from what is yet been known.

In this paragraph we give an intuitive approach. In the next paragraph (§3) we will take up this reasoning in a more expanded and in an more illustrated way, by the means of some didactic representations.

Firstly, here comes the representation of our intuitive approach:

Suppose that the Goldbach Conjecture were untrue in case of a given even number *E*.

In that case, the mirror-images (each time of course we do mean "the mirror-images throughout *E*:*2*") of all prime numbers from the left half of the number, are situated on LRwaves (for, supposing the Goldbach Conjecture being untrue, these mirror-images are always multiples of prime numbers).

In that case, the mirror-images of all prime numbers from the right half of the number, are situated on LRwaves alike (for, suppo-sing the Goldbach Conjecture being untrue, alike these mirror-images are always multi-ples of prime numbers).

This means that the mirror-images of all pri-me numbers are situated on LRwaves.

Otherwise said: in that case, the mirror-ima-ges of the multiples of the prime numbers will contain all prime numbers.

Still otherwise said: in that case, all prime numbers are situated on the mirror-images of the multiples of prime numbers.

For now:

(1°) the " multiples of prime numbers" are the LRwaves;

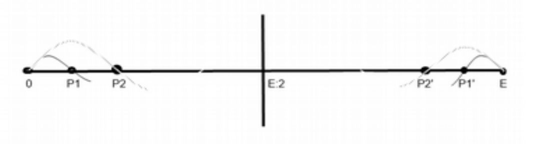
(2°) the "mirror-images of the multiples of prime numbers" are the RLwaves.

From "(1°)" and "(2°)" now follows that in that very case the LRwaves and the RLwaves necessarily coincide. For we know that the order in the rows of numbers, albeit inverted, is being conserved after the process of mirroring.

And in that very case, *E* cannot exist, as can easily be demonstrated by the means of Tschebycheff's thesis.

§3. A didactic approach

Let us consider a slip of paper. On the paper is been drawn a line-fragment, in the middle of which is drawn the point (*E*:*2*), at the outmost left the point *0*, and at the outmost right the even number *E*.



The second point from the left represents the first prime number (*P1*). From the point *0*, a black wave departs which crosses *P1* and also all of its multiples, and each time it does so on the locations at which it crosses the horizontal axis.

The third point from the left represents the second prime number (*P2*). From the point *0*, a second black wave departs which crosses *P2* and also all of its multiples, and each time it does so on the locations at which it crosses the horizontal axis.

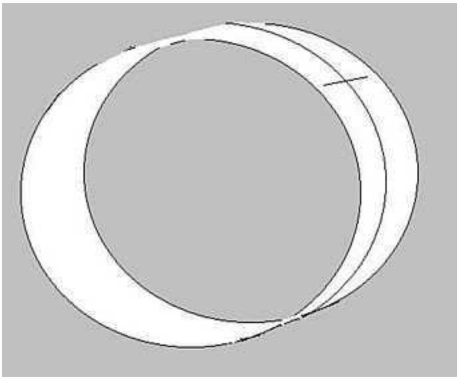
The second point from the right represents the mirror-image, throughout the mirror *E*:*2*, of the first prime number (*P1*) and is called *P1'*.

From *E* departs a wave crossing *P1'* and this wave further on crosses the horizontal axis on specific distances from *E*, which are multiples of the line-fragment *E*-*P1'*.

The third point from the right represents the mirror-image, throughout the mirror *E*:*2*, of the second prime number (*P2*) and is called *P2'*.

From *E* departs a wave crossing *P2'* and this wave further on crosses the horizontal axis on specific distances from *E*, which are multiples of the line-fragment *E*-*P2'*.

Now, we take our slip of paper, which is elastic, at its extremities, and we stretch it out... and then we fold it so that we get a string.

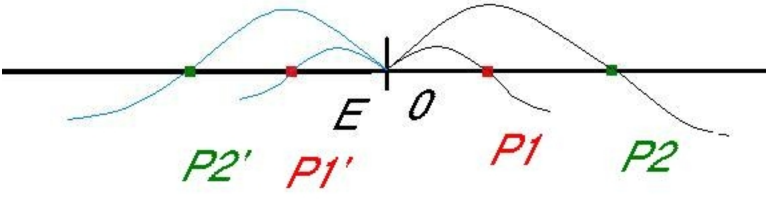


Now we tie up both the ends (the one end bearing the point *0*, the other end bearing the point *E*) to each other.

In doing so, we take care that our drawing bears the point *0* (or *E*) at the upside of the string, so that we can look upon it from above.

So, the point *E*:*2* is been situated somewhere at the southern end of the string.

Now we take a look at the string from above, and we get the following picture:



Considered in this way, it looks as if all points and waves are been mirrored throughout the point *0*, which coincides with the point *E*.

In fact, these elements mirror throughout the point *E*:*2* which is situated out of our range of vision (at the southern end), but finally we can see that this is the very same.

The prime numbers (*P1*, *P2*,...) now are been drawn at the right, and we read them from the left to the right, as well as the black LR-waves that they are constituting;

their mirror-images (*P1*', *P2*', ...) are drawn at the left, and we read them from the right to the left, alike the blue RLwaves that they are constituting.

Now we suppose that the Goldbach Conjecture were untrue, in other terms: that we found a number *E* which contradicts the Goldbach Conjecture.

The mirror-image of *P1* is *P1*'.

Due to our supposition, *P1* will be a compound number, and therefore it is situated on the black waves, which are the LRwaves.

Yet, because *P1'* is the mirror-image of *P1*, it is also situated on the blue waves, which are the RLwaves, and these blue RLwaves mirror the black LRwaves.

The mirror-image of *P2* is *P2'*.

Due to our supposition, *P2'* will be a compound number, and therefore it is situated on the LRwaves.

Yet, because *P2'* is the mirror-image of *P2*, it is also situated on the RLwaves, and these RLwaves mirror the LRwaves.

So, when, at the one hand, we should call down the prime numbers the one after the other, while following the LRwaves, we would get this row: (*P1*, *P2*, ..., ..., *P2'*, *P1'*). In doing so, it is clear that we just have to follow the string in the direction from the left to the right, first going downwards and, after having made the whole circle, coming upwards again.

When, at the one hand, we should call down the prime numbers the one after the other, while following the RLwaves, we would get this row: (*P1*', *P2*', ..., ..., *P2*, *P1*). In doing so, it is clear that we just have to follow the string in the direction from the right to the left, first going downwards and, after having made the whole circle, coming upwards again.

In doing so, we remark that not only the numbers as such, but also the order of the numbers is been mirrored.

Now, let us mirror these mirror-images, being *P1*' and *P2*', a second time throughout *E*:*2*.

We can see that the generated point *P1''* coincides with the point *P1*, and that the generated point *P2''* coincides with the point *P2*.
Now this is the case concerning all points *P''* generated in this way, because the order of these twofold mirrored numbers is been kept up.
Moreover: all numbers *P''* (which coincide with the numbers *P*) receive a supplementary attribute from the numbers *P'*:
As we have seen, the numbers *P'* are situated both on LRwaves and on RLwaves. Now then, due to the given fact that the RLwaves and LRwaves mirror each other, this also holds concerning the numbers that are been situated on it, in casu the numbers *P'*, which reflect in the numbers *P''*: the numbers *P''* are situated both on LRwaves and on RLwaves.

We know for now that the twofold mirroring keeps up the numbers as well as their mutual order.

We also know that the waves are constituted solely by the numbers involved.

So we must conclude that the LRwaves and the RLwaves necessarily do coincide.

Now, let us cut our string at the point at which we stuck it together, namely at the points *0* or *E*.

What we see now, is a drawing showing us that the black waves and the blue waves overlap each other perfectly: they coincide. The string seems to have become endless, *E*:*2* seems infinitely far away; we cannot touch it, but this is of no harm.

Now, determining that all black and blue waves coincide, this means that the black waves

(LRwaves) as well arrive at *E*. And this means that *E* must contain all prime number factors: all prime numbers must be divisors of *E*!

As a matter of fact, such a number *E* cannot exist (— it had to be infinitely big), as can be proved easily by the means of Tschebycheff's thesis. So the Goldbach Conjecture cannot be untrue. What was to be demonstrated.

A possible objection formally demonstrated to be untrue (— a start to a formal demonstration of the Goldbach Conjecture)

If the Goldbach Conjecture were untrue, it would hold that each mirror-image throughout $E:2$ of each prime number smaller than *E*, must be a compound number, which means that in that case it will hold that: $E\text{-}p_i$

(m being a natural number, $m>1$, and p_1, p_j being prime numbers).

In the case that p_i equals p_j , there is no problem, because in that case it is clear that the RLwaves and the LRwaves coincide.

Formally: if $p_i = p_j$, then from $E - p_i = mp_j$ follows that $E = mp_i + p_i = (m+1)p_i$, which means that in that case p_i is a factor of E (- in other terms: p_i is a divisor of E).

The problem rises at the point that is being objected that p_i not necessarily equals p_j.

Therefore we will now demonstrate that p_i cannot differ from p_j .

Firstly, let us give an auxiliary thesis, which says: "If $E\text{-}p_i\text{=}mp_j$, so that $\boldsymbol{p_i}$ **differs from** $\boldsymbol{p_j}$, then E may not be a multiple of p_j , so it cannot hold that: $E\text{=}bp_j$. (*b>1; b* being a natural number)".

The <u>proof of our auxiliary thesis</u> proceeds via a simple *reductio ad absurdum*, as follows: suppose that $E\text{=}p_jb$, then it follows from $E\text{-}p_i\text{=}mp_j$ that holds: $p_jb\text{-}p_i\text{=}mp_j$, hence: $p_jb\text{-}mp_j\text{=}p_i$, hence: $p_j(b\text{-}m)\text{=}p_i$, and if we state that $b\text{-}m\text{=}c$ (so that c being a natural number) then it follows: $p_jc\text{=}p_i$. In that case, the one prime number should be a multiple of the other one, which were impossible. What was to be demonstrated.

Remark. If $(b\text{-}m)=1$, then it follows that $p_i=p_j$; So it must hold that $(b\text{-}m)\neq 1$. For now, if $(b\text{-}m)\neq 1$, then p_i should be a multiple of p_j , what is impossible because prime numbers cannot be compound (i.e.: they cannot be multiples of prime numbers).

Remark that our auxiliary thesis holds concerning each prime number smaller than *E*. Further on we prove in "(*)" that this auxiliary thesis holds for each prime number smaller than *E*.

[In concrete, this means that it holds that:

if $E\text{-}p_i=m3$ then *E* cannot be a *3*-fold, so it cannot hold that $E=b3$;

if $E\text{-}p_i=m5$ then *E* cannot be a *5*-fold, so it cannot hold that $E=b5$;

if $E\text{-}p_i=m7$ then E cannot be a 7-fold, so it cannot hold that $E=b7$;
etceteras concerning all prime numbers being smaller than E.]

We remember: the Goldbach Conjecture were untrue if, for a given number E it holds that all mirror-images of the prime numbers being smaller than E would be compound, which means: if we find for each p_i that it holds that $E\text{-}p_i=mp_j$, so that $m>1$. For now, we found in our auxiliary thesis: **if p_i differs from p_j** , then E cannot be a p_j-fold. In concrete, this means that E cannot contain the factor p_j (i.e.: p_j cannot be a divisor of E) (as a matter of fact: unless $p_j=p_i$) . And this holds concerning <u>all</u> prime numbers being smaller than *E*. So, *E* will not contain any of the prime numbers smaller than *E*. So: if E-p_i=mp_j , so that p_i differs from p_j , then *E* cannot be bigger than *2*, and so **our supposition** that the Goldbach Conjecture were untrue, **fails**.

Still in question is the alternative, namely that p_i=p_j. And in that case LRwaves and RLwaves coincide.

(*) At this time there still can be some unclearness about the question **why our auxiliary thesis holds for each prime number smaller than *E*.** So, here comes the proof of our thesis, stating that **our auxiliary thesis holds concerning each prime number smaller than *E*:**

Suppose E-p_i=mp_j.

If $p_i = p_j$ then $E=(m+1)p_i$ and so it follows that E is laying on the wave of p_i.

If $p_i \neq p_j$ then $E \neq p_j b$ en and so it follows that E is not laying on the wave of p_i.

Suppose $E\text{-}p_j = np_k$ (so that p_k being a prime number and n being a natural number, $n>1$).

If $p_j = p_k$ then $E\text{-}p_j = np_j$ out of which $E=(n+1)p_j$ and then <u>E is laying on the wave of p_j</u>.

If $p_j \neq p_k$ then $E \neq np_k$ (so that n being a natural number, $n>1$) and then E is not laying on the wave of p_k.

Now, suppose $p_i \neq p_j$ and $p_j = p_k$ then follows a contradiction (see the underlined part above).

So: **either** $p_i \neq p_j$ and $p_j \neq p_k$ and in that case E is laying on a wave of p_i only, (**)

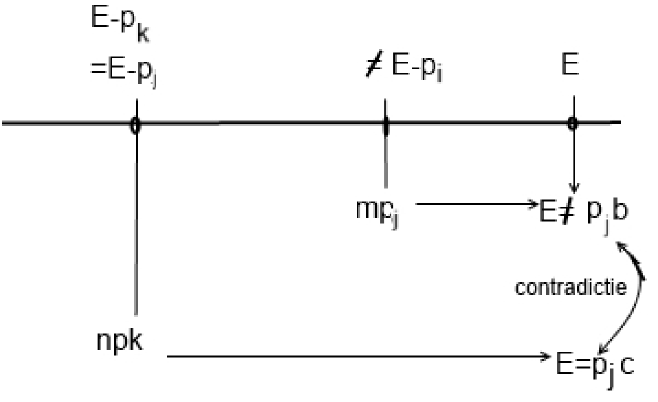
or $p_i = p_j$ and $p_j = p_k$ and then E is laying on all prime waves. (***)

Conclusion: because "(**)" is been excluded, it always holds that "(***)", which means that *all* prime-waves arrive at E and that it always holds, in $E - p_i = mp_j$, that $p_i = p_j$.

The explanation up here is been represented in a drawing at the end of this paragraph. Firstly, let us repeat this all clearly:

The image of a prime number p_i (being equal to $E - p_i$) is equal to the multiple of a prime number p_j, which means that it equals mp_j. On its turn, the image of a prime number p_j

(being equal to E-p_j) is equal to the multiple of a prime number p_k, which means that it equals np_k. And so on concerning all prime numbers smaller than E. For now, if p_i and p_j differ mutually, then, due to our auxiliary thesis, it holds that p_j cannot be a factor of E. But in that case p_j and p_k and all other prime numbers necessarily will differ mutually, because of the fact that, supposing that already p_i and p_j would differ mutually, whilst e.g. p_j would equal p_k, then a contradiction would follow. For, if, at the one hand, p_i differs from p_j , this implies that p_j cannot be a divisor of E , whilst, at the other hand, if p_j equals p_k , this implies that p_j must be a divisor of *E*. So, this contradiction makes it possible that the mirror-images of some prime numbers cannot be multiples of these prime numbers, whilst the mirror-images of other prime numbers will be multiples of these prime numbers. Hence we conclude: either all the mirror-images of the prime numbers are their own reflections, or none of them is its own multiple. The latter must be excluded because in that case *E* will not contain any of those prime number factors, and then *E* will equal either the number *2* or a number smaller than *2*. So the former possibility is left: the mirror-images of all prime numbers are necessary multiples of themselves. So far this explanation. (See also §1 for an algebraic approach). Our drawing looks like this:


E-pk
=E-pj
≠ E-pi
E
mpj
E≠ pj b
contradictie
npk
E=pj c